CONTRIBUTIONS TO MOLECULAR PHYSICS IN HIGH VACUA[1]

II.

It has been shown that the stream of molecules are shot off from the negative pole in a negatively charged condition, and their velocity is owing to the mutual repulsion between the similarly electrified pole and molecules. It became of interest to ascertain whether lateral repulsion was exerted between the molecules themselves. If the stream of molecules coming from the negative pole carried an electric current, two parallel rays should exert mutual attraction; but if nothing of the nature of an electric current was carried by the stream, it was likely that the two parallel rays would act simply as negatively electrified bodies and exert lateral repulsion. This was not difficult to put to the test of experiment.

A tube was made with two flat aluminium terminals, *a b*, close together at one end, and one terminal, *c*, at the other, as shown in Fig. 11. Along the centre of the tube, cutting the axis obliquely, is a screen of mica, painted over with a phosphorescent powder, and between the screen and the double poles, *a b*, is a disk of mica crossing the axis of the tube, and therefore nearly at right angles to the phosphorescent screen. In this mica disk are two slits—one opposite each pole *a* and *b*—running in such a direction that the molecular streams emanating from *a* and *b* when made negative shall pass through the slits, forming two horizontal sheets. These sheets striking against the oblique screen will be made evident as two horizontal lines of light. The poles *a* and *b* were somewhat bent, so that the lines of light were not quite parallel, but slightly converged. The tube being properly exhausted, the pole *a* was made negative, and *c* positive, the lower pole *b* being left idle. A sharp ray of phosphorescent light shot across the screen along the line *d f*. The negative wire was now transferred from *a* to *b*, when a ray of light shot along the screen from *e* to *f*. The two poles *a* and *b* were now connected by a wire, and the two together were made the negative pole. Two lines of light now shone on the screen, but their positions, instead of being, as before, *d f* and *e f*, were now *d g* and *e h*, as shown by the dotted lines. The wire joining the poles *a b* was removed, and the pole *a* made negative; the ray from it followed the line *d f* as before. While the coil was working, another wire hanging loose from the pole *b* was brought up to *a*, so as to make them both negative. Instantly the ray *e h* shot across the screen, and simultaneously the ray *d f* shifted its position up to *d g*. The same phenomena were observed when the pole *b* was connected with the coil, and contact was alternately made and broken with *a*; as the ray *d g* shot across, the ray *e f* dipped to *e h*.

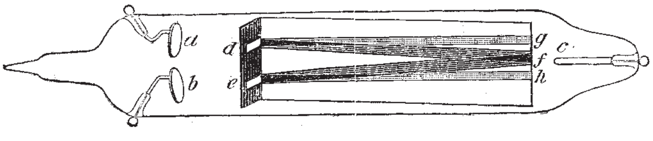

Fig. 11.

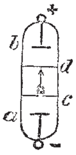

Fig. 12.

These experiments show that two parallel rays of molecules issuing from the negative pole exert lateral *repulsion*, acting like adjacent streams of similarly-electrified bodies. Had they carried an electric current they should have attracted each other, unless, indeed, the attraction in this case was not strong enough to overcome the repulsion.

Many experiments have been made to ascertain the law of the action of magnets and of wires carrying currents, on the stream of molecules.

As an indicator, a small tube, as shown in Fig. 12, was employed. The two poles are at *a* and *b*, *a* being the negative. At *c* is a plate of mica with a hole in its centre, and at *d* is a phosphorescent screen. A sharp image of the hole in the mica is projected on the centre of *d*, and the approach of a magnet causes this bright spot to move to different parts of the phosphorescent screen.

A large electro-magnet was used, actuated by two Grove's cells, and the indicator tube was carried round the magnet in different positions and the results noted. The molecular stream when under no magnetic influence passes along the axis of the tube, as shown by the small arrow (Fig. 12). It will be seen that the indicator can occupy three different directions in respect to the magnet. The magnet being held horizontally, the direction of the molecular stream may be parallel to the axis, tangential to it, or at right angles to it. In either of these positions, also, the stream may be directed one way or the other (by turning the tube round endwise). In these different positions various results are obtained which are easily illustrated with a solid model, but are somewhat complicated to explain by means of flat drawings. They are fully described in the paper.

A long tube was made similar to the small indicator shown in Fig. 12, but having a molecular trajectory six inches long. It was only exhausted to the point at which the image of the spot was just seen sharply defined on the screen, as at higher exhaustions the action of magnetism is less. The phosphorescent screen was divided into squares for convenience of noting the deflection of the spot of light. So sensitive was this to magnetic influence, that when the tube was placed parallel to the earth's equator the earth's magnetism was sufficient to cause the spot to move 5 millims. away from the position it occupied when parallel to the dipping needle (in which position the earth's magnetism did not appear to act). When held equatorially and rotated on its axis, the spot of light, being always driven in one direction independent of the rotation of the tube, appeared to travel round its normal position in a circle of 10 millims. diameter.

I have long tried to obtain continuous rotation of the molecular rays under magnetic influence, analogous to the well-known rotation obtained at lower exhaustions. Many circumstances had led me to think that such rotation could be effected. After many failures an apparatus was constructed as follows, which gave the desired results:—

A bulb (Fig. 13) was blown of German glass, and a smaller bulb was connected to each end of the larger bulb by an open, very short neck. At each extremity was a long aluminium pole projecting partly into the large bulb and turned conical at the end. After good exhaustion the passage of an induction current through this apparatus fills the centre bulb with a very fine green light, whilst the neck surrounding the pole which happens to be negative is covered with two or three dark and bright patches in constant motion, following each other round first one way and then the other, constantly changing direction and velocity, sometimes dividing into other patches, and at others fusing together into one. After a little time, probably owing to the magnetism of the earth, or that of the core of the induction coil not far off the movements sometimes become more regular, and slow rotation takes place. The patches of light concentrate into two or three, and the green light in the bulb gets more intense along two opposite lines joining the poles forming two faintly outlined patches, which slowly move round the bulb equatorially, following each other a semi-circumference apart.

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[1] "Contributions to Molecular Physics in High Vacua. Magnetic Deflection of Molecular Trajectory; Laws of Magnetic Rotation in High and Low Vacua; Phosphorogenic Properties of Molecular Discharge." By William Crookes, F.R.S. (Extracts from a paper in the *Philosophical Transactions* of the Royal Society, Part 2, 1879.) Continued from p. 104.

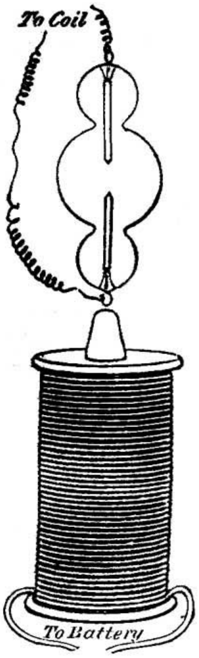


Fig. 13.

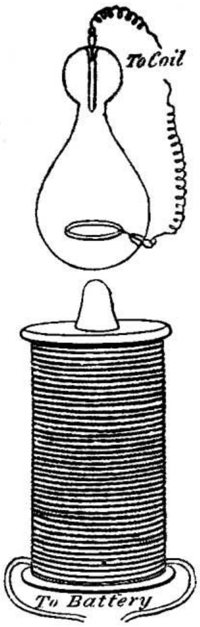
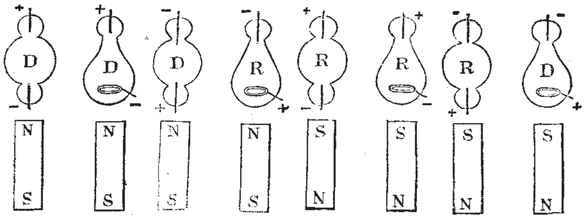


Fig. 14

An electro-magnet placed beneath in a line with the terminals (Fig. 13) converts these undecided movements into one of orderly rotation, which keeps up as long as the coil and magnet are at work.

In order to compare accurately the behaviour of the molecular streams at high exhaustions with that of the ordinary discharge through a moderately rarefied gas, another tube was taken having the upper pole an aluminium wire, and the lower one a ring, Fig. 14. It was only exhausted to such a point that the induction spark should pass freely from one pole to the other in the form of a luminous band of light, this being the form of discharge usually considered most sensitive to magnetic influence. This tube was also mounted over an electro-magnet, and the two sets of apparatus being actuated successively with the same coil and battery, the following observations were made.



Class a. Class b. Class c. Class d.

Fig. 15.

The tubes will be distinguished by the terms "high vacuum" (Fig. 13) and "low vacuum" (Fig. 14). The rotation produced in each tube will be recorded in the direction in which it would be seen by an observer above, looking vertically down on the tube, his eye being in a line with the terminals and with the axis of the magnet. When the rotation thus viewed is in the direction of the hands of a watch, it is called *direct;* the opposite movement being called *reverse.* To facilitate a clear appreciation of the actions, an outline sketch (Fig. 15) accompanies each experiment. The shape of the tube shows whether it is the high or low vacuum tube, and the letter D or R shows the direction of rotation.

a. Upper pole of electro-magnets *north.*
Induction current passing through tubes so as to make the top electrode *positive.*
Rotation in the high vacuum *direct.*
Rotation in the low vacuum *direct.*

b. Upper pole of magnets *north.*
Top electrode of tubes *negative.*
Rotation in high vacuum *direct.*
Rotation in low vacuum *reverse.*

c. Upper pole of magnets *south.*
Top electrode of tubes *positive.*
Rotation in high vacuum *reverse.*
Rotation in low vacuum *reverse.*

d. Upper pole of magnet *south.*
Top electrode of tubes *negative.*
Rotation in high vacuum *reverse.*
Rotation in low vacuum *direct.*

These experiments show that the law is not the same at high as at low exhaustions. At high exhaustions the magnet acts the same on the molecules whether they are coming to the magnet or going from it, the direction of rotation being entirely governed by the magnetic pole presented to them, as shown in cases *a* and *b* where the north pole rotates the molecular stream in a *direct* sense, although in one case the top electrode is positive and in the other negative. Cases *c* and *d* are similar; here the magnetic pole being changed, the direction of rotation changes also. The direction of rotation impressed on the molecules by a magnetic pole is opposite to the direction of the electric current circulating round the magnet.

The magnetic rotations in low vacua are not only fainter than in high vacua, but they depend as much on the direction in which the induction spark passes through the rarefied atmosphere, as upon the pole of the magnet presented to it. The luminous discharge connecting the positive and negative electrode carries a current, and the rotation is governed by the mutual action of the magnet on the perfectly flexible conductor formed by the discharge.

In high vacua, however, the law is not the same, for in cases *b* and *d* similar arrangements produce opposite rotations in high and in low vacua. The deflection exerted by a magnet on the molecular stream in a high vacuum may be compared to the action of a strong wind blowing across the line of fire from a mitrailleuse. The deflection is independent of the to-and-fro direction of the bullets, and depends entirely upon the direction of the wind.

I have already mentioned that platinum will fuse in the focus of converging molecular rays projected from a concave pole. If a brush of very fine iridio-platinum wire, which has a much higher fusing point than platinum, be used to receive the molecular bombardment, a brilliant light is produced, which might perhaps be utilised.

A piece of apparatus was constructed in which a plate of German glass was held in the focus of the molecular bombardment. The vacuum was so good that no hydrogen or other lines could be seen in the spectrum of the emitted light. The focus was now allowed to play on the glass, when the glass soon became red hot. Gas appeared in the tube, and hydrogen lines now were visible in the spectrum. The gas was pumped out until hydrogen disappeared from the spectrum. It was now possible to heat the glass to dull redness without hydrogen coming in the tube; but as soon as the heat approached the fusing point the characteristic lines appeared. It was found that however highly I heated the glass and then pumped the tube free from hydrogen, I had only to heat the glass to a still higher temperature to get a hydrogen spectrum in the tube. I consider the hydrogen comes from vapour of water, which is obstinately held in the superficial pores, and which is not entirely driven off by anything short of actual fusion of the glass. The bubbles noticed when the disintegrated and fused surface of the tube was examined under the microscope are probably caused by escaping vapour of water.

When the negative discharge has been playing for some time on German glass, so as to render it strongly phosphorescent, the intensity of glow gradually diminishes. Some of this decline is due to the heating of the glass or to some other temporary action, for the glass partially recovers its property after rest; some is due to a superficial change of the surface of the glass; but part of the diminished sensitiveness is due to the surface of the glass becoming coated with this brown stain.

The luminous image of a hole in a plate of mica was projected from a *platinum* plate used as a negative pole, to the side of a glass bulb. The coil was kept playing for some time until the inside of the bulb was thoroughly darkened by projected platinum. Although a bundle of molecular rays could be seen all the time passing from the platinum through the hole in the mica to the glass, where it shone with a bright green light, I could detect no trace of extra darkening when the part of the glass formerly occupied by the green spot was carefully examined. Platinum is a metal which flies off in a remarkable manner when it forms the negative pole. It therefore appears from this experiment that the molecular stream does not consist of particles of the negative pole shot off from it.

One of the most striking of the phenomena attending this research has been the remarkable power which the molecular rays in a high vacuum possess of causing phosphorescence in bodies on which they fall. Substances known to be phosphorescent under ordinary circumstances shine with great splendour when subjected to the negative discharge in a high vacuum. Thus, a preparation of sulphide of calcium, much used now in Paris for coating clock faces which remain luminous after dark, is invaluable in these researches for the preparation of phosphorescent screens whereon to trace the paths and trajectories of the molecules. It shines with a bright blue-violet light, and when on a surface of several square inches is sufficient to light up a room. Modifications of these phosphorescent sulphides shine with a yellow, orange, and green light.

The only body I have yet met with which surpasses the luminous sulphides both in brilliancy and variety of colour is the diamond. Most of these gems, whether cut or in the rough, when coming from the South African fields, phosphoresce of a brilliant light blue colour. Diamonds from other localities shine with different colours, such as bright blue, pale blue, apricot, red, yellowish-green, orange, and bright green. One beautiful green diamond in my collection when phosphorescing in a good vacuum gives almost as much light as a candle; the light is pale green—almost white. A beautiful collection of diamond crystals kindly lent me by Prof. Maskelyne phosphoresce with nearly all the colours of the rainbow, the different faces glowing with different shades of colour.

Next to the diamond, alumina in the form of ruby is perhaps the most strikingly phosphorescent stone I have examined. It glows with a rich, full red; and a remarkable feature is that it is of little consequence what degree of colour the earth or stone possesses naturally, the colour of the phosphorescence is nearly the same in all cases; chemically precipitated amorphous alumina, rubies of a pale reddish-yellow, and gems of the prized "pigeon's blood" colour, glowing alike in the vacuum, thus corroborating E. Becquerel's results on the action of light on alumina and its compounds in the phosphoroscope (*Annales de Chimie et de Physique*, ser. 3, vol. lvii.). Nothing can be more beautiful than the effect presented by a mass of rough rubies when the molecular discharge plays on them in a high vacuum. They glow as if they were red hot, and the illuminating effect is almost equal to that of the diamond under similar circumstances.

By the kindness of M. Ch. Feil, who has placed large masses of his artificial ruby crystals at my service, I have been enabled to compare the behaviour of the artificially formed crystals with that of the natural ruby. In the vacuum there is no difference whatever; the colour of the phosphorescence emitted by M. Feil's crystals is of just as intense a colour, and quite as pure in character, as that given by the natural stone. This affords another proof, if one were needed, that Messrs. Fremy and Feil have actually succeeded in the artificial formation of the veritable ruby, and have not simply obtained crystals which imitate it in hardness and colour.

The appearance of the alumina glow in the spectroscope is remarkable. There is a faint continuous spectrum ending in the red somewhere near the line B; then a black space, and next an intensely brilliant and sharp red line to which nearly the whole of the intensity of the coloured glow is due. The wave-length of this red line, which appears characteristic of this form of alumina, is 689·5 m.m.m., as near as I can measure in my spectroscope; the maximum probable error being about ± ·3.

This line coincides with the one described by E. Becquerel as being the most brilliant of the lines in the spectrum of the light of alumina, in its various forms, when glowing in the phosphoroscope.

This coincidence affords a good proof of the identity of the phosphorescent light, whether the phosphorescence be produced by radiation, as in Becquerel's experiments, or by molecular impact in a high vacuum.

I have been favoured by my friend Prof. Maskelyne with the following notes of results obtained on submitting to the molecular discharge various crystals which he lent me for the purpose of these experiments:—

"Diamond crystals. A very small crystal, exhibiting large cube faces with the edges and angles truncated, was of a rich apricot colour, the dodecahedral faces of a clear yellow, and the octahedral of another yellow tint. No polarisation of the light was detected. Some were opaque; some gave a bluish hazy light.

"Emerald. A small hexagonal prism gave out a fine crimson-red colour. The light was polarised, apparently completely, in a plane perpendicular to the axis; this would correspond therefore to extraordinary rays which in emerald, as a negative crystal, represent the quicker rays vibrating presumably parallel to the optic axis of the crystal.

"Other emeralds behaved in the same way, though the illumination in two others experimented with appeared confined more particularly to one end—the end opposite to that at which the crystals presented some (in one instance fine) terminal faces.

"Beryls exhibited no corresponding phenomena.

"Sapphires gave out a bluish-grey light, distinctly polarised in a plane perpendicular to the axis. In this case, again, the ray developed corresponds to the extraordinary or quicker ray.

"Ruby gives out a transcendently fine crimson colour, exhibiting no marked distinction in the plane of its polarisation, though in one part of a stone the colour was extinguished by a Nicol prism with its long diagonal parallel to the axis of the crystal. Here, therefore, also the light was that of the extraordinary ray.

"It seemed desirable to determine the nature of the phenomena in the case of positive crystals, and accordingly crystals of quartz, phenakite, tinstone, and hyacinth (zircon), were placed in a tube and experimented on.

"The only crystals that gave definite results were tintone and hyacinth. A small crystal of the former mineral glowed with a fine yellow light, which was extinguished almost entirely when the long diagonal of the Nicol was perpendicular to the axis of the crystal.

"Here, therefore, the plane of polarisation of the emitted light was parallel to the axis of the crystal, and here it is again the quicker, though in this case (of an optically positive crystal) it is the ordinary ray which corresponds to the light evoked by the electric stream.

"So far, then, the experiments accord with the quicker vibrations being called into play, and therefore in a negative crystal the extraordinary and in a positive crystal the ordinary is the ray evoked.

"A crystal of hyacinth, however, introduced a new phenomenon. In this optically positive crystal the ordinary ray was of a pale pink hue, the extraordinary of a very beautiful lavender-blue colour. In another crystal, like the former from Expailly, the ordinary ray was of a pale blue, the extraordinary of a deep violet. A large crystal from Ceylon gave the ordinary ray of a yellow colour, the extraordinary ray of a deep violet hue.

"Several other substances were experimented on, including some that are remarkable for optical properties, among which were tourmaline, andalusite, enstatite, minerals of the augite class, apatite, topaz, chrysoberyl, peridot, garnets of various kinds, and parisite. So far, however, these minerals have given no result, and it will be seen that the crystals which have thus far given out light in any remarkable degree are, besides diamond, uniaxal crystals (an anomaly not likely to be sustained by further experiment); and the only conclusion arrived at is, that the rays whose direction of vibration corresponds to the direction of maximum optical elasticity in the crystal are always originated where any light is given out. As yet, however, the induction on which so remarkable a principle is suggested cannot be considered sufficiently extended to justify that principle being accepted as other than probable."

WILLIAM CROOKES

ON THE LAW OF FATIGUE IN THE WORK DONE BY MEN OR ANIMALS

THE Rev. Dr. Haughton, of Trinity College, Dublin, has recently brought to a conclusion a series of papers on Animal Mechanics published in the *Proceedings* of the Royal Society. The ninth of these papers was appointed the Croonian Lecture for the present year, and the tenth paper closes the series.

The most important subject involved in these papers is the experimental determination of the law that regulates fatigue in men and animals, when work is done, so as to bring on fatigue.

Many writers, such as Bouguer, Euler, and others, have laid down mathematical formulæ, connecting the force overcome with the velocity of the movement; but these theoretical speculations have never received the assent of practical engineers.

Venturoli points out a method of observations and experiments which would serve to determine the form of the function which expresses the force in terms of the velocity, after which a few carefully planned experiments would determine the constant coefficients; and he adds that "such a discovery would be of the greatest usefulness to the science of mechanics, upon which it depends, how to employ, to the greatest possible advantage, the force of animal agents."

Dr. Haughton believes that he has found the proper form of this function, by means of experiments, and sums it up in what he calls the *Law of Fatigue*, which he thus expresses:—

The product of the total work done by the rate of work is constant, at the time when fatigue stops the work.

If W denote the total work done, the law of fatigue gives us—

$$W \frac{dW}{dt} = \text{const.}$$

or

$$\frac{W^2}{T} = \text{const.} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The experiments made by Dr. Haughton from 1875 to 1880 consisted chiefly in lifting or holding various weights by means of the arms; the law of fatigue giving, in each case, an appropriate equation, with which the results of the experiments were compared. When the experiments consisted in raising weights on the outstretched arms, at fixed rates, the law of fatigue gave the following expression—

$$(w + a)^2 n = A \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where w, n, are the weight held in the hand, and the number of times it is lifted, A is a constant to be determined by experiment, and a another constant depending on the weight of the limb and its appendages.

The equation (2) represents a cubical hyperbola.

The *useful work* done is represented by the equation—

$$w\,n = \frac{A\,w}{(w + a)^2} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This denotes a cuspidal cubic, and the *useful* work is a maximum, when $w = a$, or the weight used is equal to the constant depending on the weight of the limb and its appendages.

When the weights were lowered as well as raised at fixed rates, and no rest at all permitted, the law of fatigue became—

$$\frac{n\,(1 + \beta^2 t^2)}{t} = A \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where n, t, are the number and time of lift, A is a constant depending on experiment, and β is a constant involving the time of lift (τ) at which the *maximum* work is done.

Equation (4) denotes a cuspidal cubic.

When the weights are held on the palms of the outstretched hands, until the experiment is stopped by fatigue, the law becomes—

$$(w + a)^2 t = A \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where t is the whole time of holding out.

This equation denotes a cubical hyperbola.

The *Law of Fatigue* seems, in itself, probable enough, but of course its real value depends on its agreement with the results of experiment.

If W denote the total work done and R the rate of work, the law becomes, simply—

$$W \times R = \text{const.} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

If different limbs, or animals were used, each working in its own way, and under its own conditions, the *Law of Fatigue* would become—

$$W R = W_1 R_1 + W_2 R_2 + W_3 R_3 + \&c \quad . \quad (7)$$

and the problem for the engineer would be, so to arrange the work and rate of work of each agent employed, as to make the *useful work* a maximum, the work both useful and not useful, in all its parts, remaining subject to the conditions imposed by equation (7).

In using equation (5) in his concluding paper, detailing the results of experiments made on Dr. Alexander Macalister, Dr. Haughton treats a as an unknown quantity, and finds from all the observations its most probable value to be—

$$a = 5{\cdot}68 \text{ lbs.}$$

This result was compared with that of direct measurements made on Dr. Macalister himself, and indirect measurements made on the dead subject, from all of which Dr. Haughton concluded the value of a to be—